

Chapter 1  
Section 1.6

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**Equations Involving Square Roots**

In general, it is easier to work with an equation not containing square roots. This is why we use the formula  $a^2 + b^2 = c^2$  instead of  $\sqrt{a^2 + b^2} = c$ . Therefore, in most instances, we will try squaring both sides of an equation involving square roots. **Hint:** It is usually easier to isolate the square root first.

- (1) By isolating the square root and squaring both sides, solve  $\sqrt{x} + 12 = x$ .

~~scribble~~  $x = 9 \text{ or } 16$

- (2) Solve  $w = \frac{\sqrt{1-3w}}{2}$ .

$w = -1 \text{ or } \frac{1}{4}$

- (3) By squaring each side twice, solve  $\sqrt{n+4} + \sqrt{n-1} = 5$ .

$n = 5$

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**Equations of Quadratic Type**

**Def:** An equation of quadratic type is an equation of the form  $au^2 + bu + c = 0$  where  $u$  is an algebraic expression.

**Q:** Is a quadratic equation an example of an equation of quadratic type? *Yes*

**Exercises**

- (1) Solve the following equation of quadratic type  $x^4 - 14x^2 + 45 = 0$ .

$x = \pm 3 \text{ or } \pm \sqrt{5}$

- (2) Solve the equation  $(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$ .

$x = 1, \pm 2, -3$

- (3) Solve this equation of quadratic type  $x^{2/3} - 9x^{1/3} + 8 = 0$ .

$x = 1, 512$

### Equations with Rational Exponents

Solving an equation with rational exponents involves a similar method to solving an equation with square roots (since a square root is an exponent of  $1/2$  which is definitely rational). You first want to isolate your term with rational exponent and then raise both sides to the reciprocal of the rational exponent. **Hint:** Remember our exponent rules,  $x^{a/b} = (x^{1/b})^a = (x^a)^{1/b}$ .

(1) Solve the equation  $x^{4/3} = 625$ .

$$x = 125$$

(2) Solve the equation  $(y - 2)^{-5/2} = 32$ .

$$y = \frac{9}{4}$$

(3) If you had troubles with (3) of the previous section, now is the time to go back and finish it.

**Q:** When dealing with rational equations  $x^{a/b} = c$  with  $a, b, c \in \mathbf{R}$  and  $b \neq 0$  when will your answer require a  $\pm$  symbol?

When  $a$  (the numerator) is even, B/C then  $(x)^a = (-x)^a$ .

### Equations Involving Absolute Value

We have already discussed methods for solving simple absolute value equations. When you have an equation of the form  $|u| = |v|$  where  $u$  and  $v$  are algebraic expressions you just need to remember that this is equivalent to saying  $u$  and  $v$  are either equal or opposite.

(1) Solve the equation  $|x^2 - 6| = 5x$ .

$$x = \pm 1, \pm 6$$

(2) Solve the equation  $|x^2 - 2x| = |3x - 6|$ .

$$x = 2, \pm 3$$